

Calculation of AC Losses in HTS Tape With FEA Program ANSYS

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Abstract—The commercially available finite element analysis program ANSYS, primarily designed for solving problems for ferromagnetic materials and conventional conductors, has been used for simulating the ac losses in HTS tape carrying transport current or exposed to a perpendicular field. Details on how to develop the program as a calculation tool for HTS materials are presented; in particular some factors related to the numerical accuracy and efficiency are discussed. The approach is based on a 2D electromagnetic transient analysis, where the conductivity of the HTS material is simulated in terms of a power law. The commonly used flux creep model and Bean model can be realized by adjusting the power law index value from 2 to ∞ . For the Bean model, the simulated results are verified by comparison with the analytical formulas given by Norris, which shows the error to be less than 0.5%. For the flux creep model, the accuracy is estimated by comparison with Brandt's method for a basic cylinder geometry, showing a good agreement.

Index Terms—Ac loss, ANSYS, simulation software.

I. INTRODUCTION

THE ac loss of high temperature superconductor (HTS) tapes is regarded as one of the key issues limiting the large-scale application of HTS tapes in ac electrical devices, such as transformers, motors and cables. Effective evaluation of ac losses and methods for the reduction of ac losses have been studied intensively for many years. The numerical simulation of ac losses is, on one level, probably more important than experimental methods, because the results can give ac loss prediction to the cases where fabrication techniques are currently still not available, and thus guide the ac loss reduction work. The basic idea for the calculation of ac loss is to take the HTS tape as a conductor that has a power law $E - J$ relation. This relation is substituted into the Maxwell equations and subjected to certain boundary conditions to determine the solution.

There are several methods for solving ac losses. The Finite Difference Analysis (FDA) [1], [2] and Finite Element Analysis (FEA) [3] are two commonly used methods, which have been used for many years. The alternative method of Brandt is an effective method for 2D calculation, in which a Green function closely related to the sample geometry is introduced. The main advantage of Brandt's method over FDA and FEA method is that it only takes the sample region into consideration. The calculation details for a rectangular bar in perpendicular field are reported in [4] and its extension to transport losses is re-

ported in [5]. Recent years, FEA packages such as FLUX2D and FLUX3D have also been widely used. The powerful pre-process function as well as the ability to deal with the nonlinear $E - J$ relation, not only saves time in coding but also enables complex geometry [6] or even 3D model to be used [7].

In this paper, an alternative method to calculate ac losses of HTS tape is reported. The method is based on re-developing a commercially available FEA package ANSYS [8], which was once popularly used in solving problems of ferromagnetic materials. The details on how to develop the package as a calculation tool for HTS material are reported in Section II. Applications for samples carrying transport current in relation to the Bean model and flux creep model are introduced in Section III. Some concerns on the calculation efficiency and accuracy are also discussed.

II. THEORETICAL BACKGROUND

A. Formulation

The basic electromagnetic equation for dynamic analysis uses a magnetic vector potential \mathbf{A} in conjunction with a time-integrated electric scale potential ν

$$\frac{1}{\mu_0} \nabla^2 \vec{A} + \frac{1}{\rho} \frac{\partial \vec{A}}{\partial t} = -\frac{1}{\rho} \frac{\partial \nabla \nu}{\partial t}. \quad (1)$$

The introduction of ν is only required for current carrying cases. For a sample with infinite length in the Z direction, the magnetic vector \mathbf{A} and ν only have Z component and ν is constant over the conductor cross section [9]. The current density is derived from \mathbf{A} and ν

$$\vec{J} = -\frac{1}{\rho} \frac{\partial \vec{A}}{\partial t} - \frac{1}{\rho} \frac{\partial (\nabla \nu)}{\partial t}. \quad (2)$$

The equivalent resistivity of a HTS tape is expressed by a power law $E - J$ relation as

$$\rho = \frac{E_c}{J_c} \left(\frac{J}{J_c} \right)^{n-1} \quad (3)$$

The n value for a typical Bi2223/Ag multi-filamentary tape is around 10.

B. Adapting to ANSYS

Let us consider a rectangular bar bounded by $2a \times 2b$ exposed to perpendicular field. The rectangular bar is assumed to be infinitely long, thus the magnetic vector potential \mathbf{A} has only Z component. The model is built in quarter symmetry with flux-parallel ($\mathbf{A} = 0$) at $x = 0$ and flux normal boundary con-

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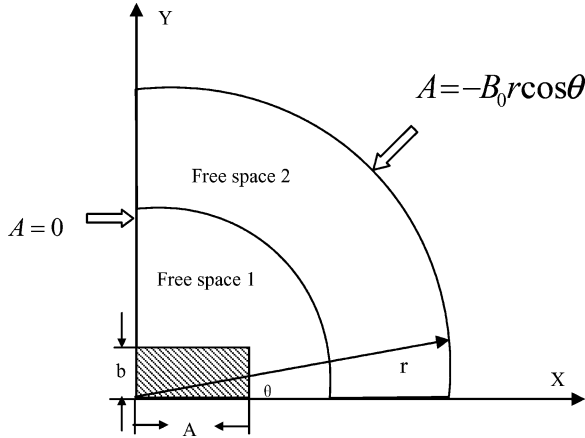


Fig. 1. Boundary condition of rectangular bar exposed to perpendicular field.

dition at $y = 0$ naturally occurring. As can be seen in Fig. 1, the outermost circumference boundary is set at r , where r is normally 100 times larger than b . An external magnetic field is achieved by applying \mathbf{A} at the boundary. The magnitude, as well as, the direction of the field is determined by $\nabla \times \mathbf{A} = \mathbf{B}$ in combination with boundary condition. For instance, a uniform perpendicular field is expressed at the circular boundary as $\mathbf{A} = -B_0 r \cos \theta$, where B_0 is the magnitude of the field, r is the radius of the outermost circular boundary and θ is the angle between r and the x axis. In general, we set two free spaces. The elements adjacent to the sample region in free space 1 require fine meshing, which guarantees a smooth transition across the interface.

For rectangular bar carrying transport current, the flux normal condition is applied to both at symmetric boundaries $x = 0$ and $y = 0$. The total current I is implemented using a boundary condition in terms of the Ampere law, $I = (1/\mu_0) \int \nabla \times \mathbf{A} \cdot d\mathbf{l}$. Alternatively, the current flow can be achieved by the F command provided within ANSYS.

Solving (1) for a constant ρ is one of the basic functions provided by ANSYS 2D transient analysis. The calculation starts off by attributing a constant ρ to the entire sample region, which is meshed in advance. Next a set of J solutions in each element under an initial exciting current or field is determined. For each new sub step, ρ is adjusted iteratively according to (3) until ρ in each element satisfies

$$\frac{|\rho^{k+1} - \rho^k|}{\rho^k} \leq \xi \quad (4)$$

where ξ is a convergence criterion to stop the iteration; $\xi = 0.001$ in one case. Principally, (1)–(3) compose of a set of nonlinear system equations. There are no analytically superior techniques for approximation of the solution to these systems. In fact, the way the ρ adjusted toward its final value is very sensitive to the model we use. The discussion of this with respect to different models, i.e. Bean model or flux creep model will be left to the following section. Technically, the computational process for (4) is also time consuming. Hence, a constant iteration number is always set at the beginning of iteration. This iteration number is highly sensitive to the n value and to the speed of the field variation, normally ranging from 50–160. All

re-developing code has been done with the ANSYS Parametric Design Language (APDL).

III. RESULT AND DISCUSSING

A. Flux Creep Model

As mentioned in the last section, there is no analytical answer to the nonlinear system. For the flux creep model, the successive over relaxation (SOR) technique is one of the most commonly used methods. This technique is expressed by the formula as

$$\rho^{k+1} = \rho^k + \beta(\rho^{k+1} - \rho^k) \quad (5)$$

where β is the over relaxation factor, typically equal to 0.1. The β value can partly compensate the intensive ρ variation between each k and $k + 1$ steps owing to the power law $E - J$ relation. If n value is too big, we can decrease the β further but at the same time this will increase the number of iteration steps to convergence. Optionally, if n value is larger than 25, the (5) can be closely followed by

$$\rho^{k+1} = \rho^k \sqrt{\frac{\rho^{k+1}}{\rho^k}}. \quad (6)$$

This choice provides optimum computational stability and speed. Another way to compensate the large n effect is to set a minimum constant resistivity, here we assign $1 \times 10^{-16} \Omega \cdot \text{m}$, to every element, i.e. even when the current density equals zero, the local resistivity cannot be smaller than $1 \times 10^{-16} \Omega \cdot \text{m}$. To compensate for an initial effect, a time-stepping procedure should be used. In our calculation, two time cycles are sufficient for reaching a sinusoidal steady state. Since in each cycle, at least 32 time steps should be used, the analysis requires 64 nonlinear solution steps. The determination of the iteration number in each nonlinear solution step, as mentioned before depends on many factors. There is no general answer to it, but experience has shown that the higher the n value and the lower the frequency, the more convergence steps are required. The largest n value we have tried is 35, for a frequency of 5 Hz. Above this value the calculation becomes severely unstable and computationally intensive. If J in each element is known, the total ac loss is obtained by integration over the last 1/4 cycle T over the conductor cross section S as

$$Q = 4 \int_T \int_S \rho \cdot |\mathbf{J}|^2 ds dt \quad (7)$$

where ρ is given in (3).

A significant weakness of the calculation method lies in the requirement that at each iteration the calculation has to restart from previous result files; this is not general, but is required in the ANSYS Single Restart Procedure. This results in an unavoidable data transfer between the harddisk and memory, and thus slows down the calculation efficiency. So far, the application has only been realized in an infinite long cylinder carrying transport current. In this case, A has only z component which dependent on the radius only. As can be seen in Fig. 2, a simplified 3° slice representing 1/120 of total cylinder is built. The free

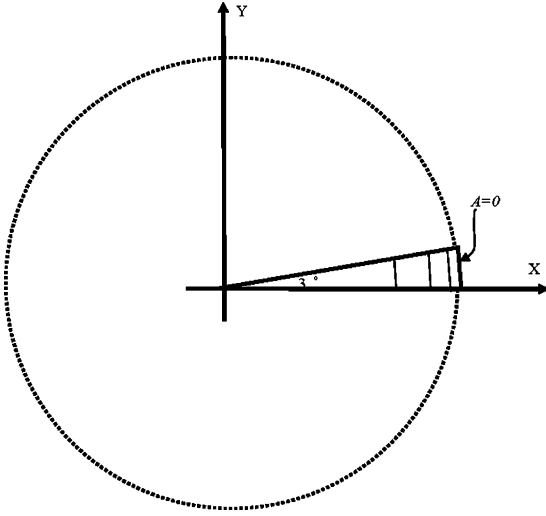


Fig. 2. A 3° slice model, representing 1/120 of a total cylinder carrying transport current.

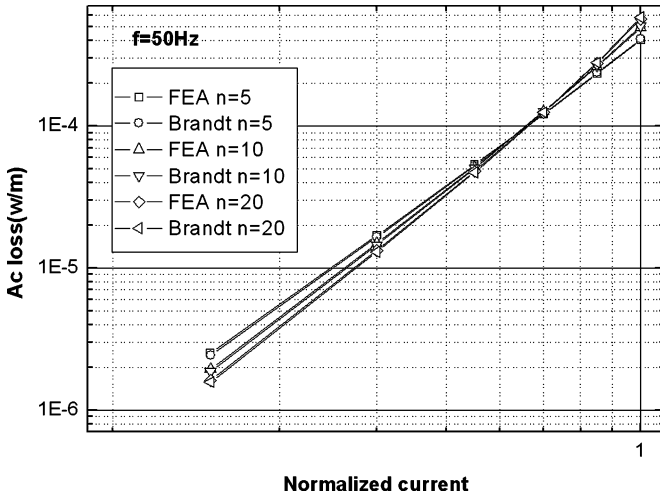


Fig. 3. Comparison of FEA results to Brandt model for the ac losses as function of a transport current for different n values.

space outside the sample can also be ignored. Two hundred elements with same area are meshed along the radial direction. The final results is obtained by multiplying a factor of 120 of total losses. On a computer equipped with a 2.8 GHz processor, the averaged time for a 2 cycle calculation, i.e. one current point, is less than 2 hour. Since there is no analytic solution for this problem, the calculation accuracy can only be checked by comparison with the Brandt method. Results of the ac loss for $n = 5, 10, \text{ and } 20$, using a 50 Hz exciting current are presented in Fig. 3. The ac losses calculated by the FEA method are always smaller than the Brandt method in the low current region. In the high current region, on the contrary, the ac losses calculated by the FEA method are smaller than calculated using Brandt method. The biggest divergence between these two methods is in the low current region with a value of less than 5%.

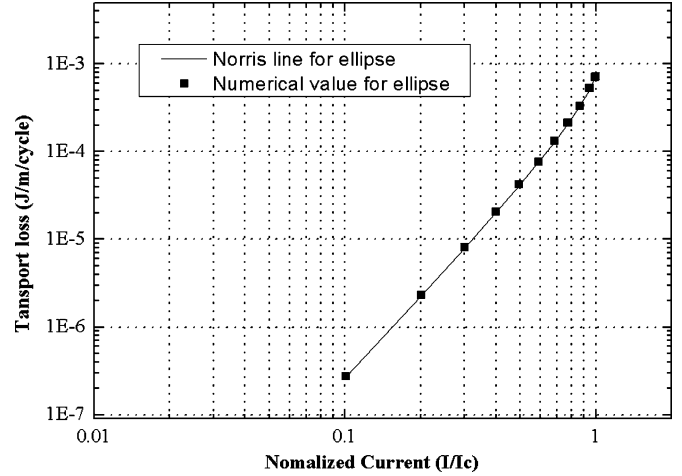


Fig. 4. Comparison of transport loss in an elliptical section to Norris elliptical model.

B. Bean Model

Although the Bean model, corresponding to $n \rightarrow \infty$ in (3), seems difficulty to solve, the $J=\text{constant}$ property, actually, results in a facilitation of calculation. For the Bean model, the ρ^{k+1} step can be determined directly in a simple way as

$$\rho^{k+1} = \rho^k \frac{J^k}{J_c} \quad (8)$$

We have found that at most 40 iteration steps are enough to reach a convergence criterion of $\xi = 0.001$ in (4). Moreover, the history independence characteristic of the problem makes it possible to simulate only 1/4 of a total cycle i.e. from $0-H_{\text{peak}}$ or $0-I_{\text{peak}}$, no matter how quickly the field varies. An even more important advantage is that there is no need to save and resume a file between the harddisk and memory. The calculation efficiency is therefore improved greatly.

For a sample exposed to perpendicular field. The total loss in a sample is determined by integrating the $M(B_a)$ loop as

$$Q = \frac{1}{\mu_0} \int_{\text{cycle}} \vec{M} d\vec{B} \quad (9)$$

where M is the magnetization of the sample. From the first 1/4 magnetization curve, the complete magnetization loop in a cycled applied field with amplitude B_{peak} is obtained as in [12] by

$$\begin{aligned} \vec{M}_{\uparrow}(\vec{B}_a) &= \left(\frac{1}{\mu_0}\right) \vec{M}(\vec{B}_{\text{peak}}) - \left(\frac{2}{\mu_0}\right) \vec{M} \left[(\vec{B}_{\text{peak}} - \vec{B}_a) \right] \\ \vec{M}_{\downarrow}(\vec{B}_a) &= -\vec{M}_{\uparrow}(-\vec{B}_a) \end{aligned} \quad (10)$$

where M_{\uparrow} and M_{\downarrow} are the magnetization in decreasing and increasing \mathbf{B}_a .

For a current carrying conductor, if J in each element is determined, the loss in total cycle is followed using a method proposed by Norris [11] as

$$W = \int 4J\phi da \quad (11)$$

where ϕ is the flux enclosed between sample center and elements and where the da is the area of each element. The accuracy of the calculation is checked by comparison with the classical Norris elliptic curve. For this purpose, a quarter ellipse as well as necessary free space was built using 2000 elements with the same area uniformly distributed in the sample region. As can be seen in Fig. 4, the maximum error is estimated to be less than 0.5%. The computation time for a full curve is within 3 hours.

The significant calculation capacity in Bean model makes it possible for us to build relatively complex model such as multi-filamentary tape, where we hope to refine the element in every filament and to build 3D models where the number of the element is always 100 times larger than required for a 2D model.

IV. CONCLUSION

A flexible method to simulate the ac loss of HTS tape is achieved using the commercial FEA package ANSYS. The method allows both the flux creep and Bean model to be taken into consideration, and is valid for the current carrying and field exposing cases. For the flux creep model, the comparison between the FEA results and the results obtained by Brandt method shows a good agreement. However, at proved accuracy, the flux creep model is only applicable for a simple 1D case. For the Bean model, the numerical error is estimated to be less than 0.5% by comparison with the theoretical value given by Norris. The calculation efficiency of the Bean model shows a great potential to handling for complex 2D or even 3D models in the following study.

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